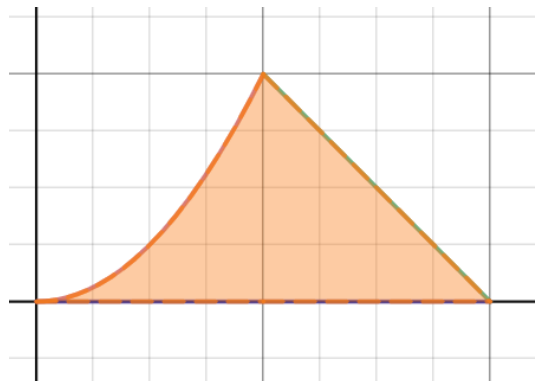


Calculus II: Final Exam

Instructions: When possible, supporting work must be shown to receive credit for each problem.

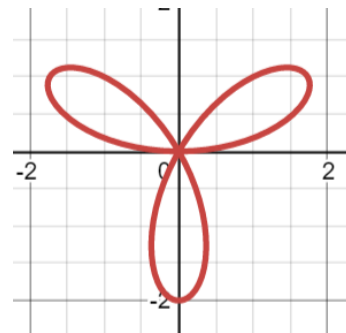
1. (10 points) Let R represent the region in the xy -plane bounded by the graphs of the following equations: $x = \sqrt{2y}$, $x = 4 - y$, and $y = 0$. **Set up** an integral representing the volume of the solid formed by rotating the region R about each axis listed below: **(You do not need to evaluate these integrals.)**

- a. x -axis
- b. y -axis
- c. the horizontal line $y = -1$



2. (5 points) Write a definite integral that gives the area of the region bounded by $x = \sqrt{2y}$, $x = 4 - y$, and $y = 0$ (see the figure in problem 1, above). **You do not need to evaluate the integral.**
3. (5 points) Differentiate the function $f(x) = x^4 \arcsin(5x^2)$. You **do not** need to simplify your answer.
4. (6 points) Evaluate the integral: $\int \frac{7x-2}{(2x-1)(x+1)} dx$
5. (5 points) Evaluate the integral: $\int \frac{e^x}{e^{2x}+1} dx$
6. (6 points) Evaluate the integral: $\int \frac{1}{x^2\sqrt{x^2-25}} dx$. Hint: Use trigonometric substitution.
7. (5 points) Evaluate the integral: $\int (4x - 3)e^x dx$
8. (5 points) Evaluate the integral: $\int \sin^3 x \cos^2 x dx$
9. (5 points) Consider the improper integral $\int_1^\infty \frac{1}{x^4} dx$. First, express the improper integral as a limit, then determine whether it converges or diverges. If it converges, state its value.
10. (5 points) Evaluate the limit: $\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{2x}$. Please write out all steps showing how you resolve the indeterminate form.
11. (5 points) Determine whether the **sequence** $\{a_n = \frac{6n^3+7}{n^3}\}$ converges or diverges.

12. (5 points) Determine if the series $\sum_{n=1}^{\infty} \frac{4n}{\sqrt{n^3} + 5}$ converges or diverges. Indicate the convergence test used.
13. (5 points) Determine if the series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ converges or diverges. Indicate the convergence test used.
14. (6 points) Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n-1}$ **absolutely converges, conditionally converges, or diverges**. Indicate the convergence test(s) used.
15. (5 points) Find the **radius of convergence** for the power series $\sum_{n=1}^{\infty} \frac{1}{n4^n} (x-3)^n$. (You do not need to find the interval of convergence, but show your work and indicate the convergence test used.)
16. (a) (3 points) Change the given rectangular equation to polar form: $x^2 + y^2 = 3y$
- (b) (3 points) Change the given polar equation to rectangular form: $r = 5 \cos \theta$
17. (5 points) Find the area of **one petal** of the three-leaf rose $r = 2 \sin 3\theta$.



18. Let $f(x) = \ln x$. Recall that the Taylor Series is given by: $\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!} x^n$.
- a. (5 points) Find the fourth Taylor polynomial for f , centered at 1.
- b. (3 points) Write the Taylor **series** (the infinite series, expressed in sigma notation) for $f(x) = \ln x$, centered at 1.